The Shape of the Distribution of Price Changes in Egypt using Robust Measures of Skewness and Kurtosis

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ABSTRACT

This paper introduces old and new robust measures of skewness and kurtosis that have been developed in the statistics literature. Some of these measures are used to study the shape of the distribution of the Egyptian consumer price changes over the period January 1995 – May 2005. It was found that the price distribution is asymmetric with little right skewness and mild excess kurtosis. Moreover, the application shows the nonrobustness of the classical measures and that the robust measures are not heavily influenced by outliers as the classical measures.

Keywords: skewness, kurtosis, quantiles, robustness, tail weight, distribution of price changes.

1. Introduction

The shape of a probability distribution is often summarized by the distribution's skewness and kurtosis. Skewness describes the asymmetry of a distribution. A symmetric distribution has zero skewness, an asymmetric distribution with the largest tail to the right has positive skewness, and a distribution with a longer left tail has negative skewness. The skewness of a univariate data set sampled from a continuous distribution is measured by the standardized third central moment. It is defined as:

$$\beta_1 = \mu_3 / \mu_2^{3/2} \qquad (1.1)$$

Kurtosis is measured by the standardized fourth central moment. It is defined as:

$$\beta_2 = \mu_4 / \mu_2^{2} \qquad (1.2)$$

The normal distribution with a value of β_2 equals 3 is used as a standard, and $\beta_2 - 3$ is called the coefficient of excess of the distribution.

If $\beta_2 < 3$, the distribution is called platykurtic, which is flat topped compared with the normal. If $\beta_2 > 3$, the distribution is called leptokurtic, which is more sharply peaked than the normal, and if $\beta_2 = 3$, the distribution is called mesokurtic, having a shape comparable to that of the normal.

Although moments play an important role in statistical inference, they are very poor indicators of distributional shape (Balanda & MacGillivray, 1988). It is well known that the sample moments are sensitive to outliers, and since β_1 and β_2 are based on sample moments, they are also sensitive to outliers. Moreover, the impact of outliers is greatly amplified because they are raised to the third and fourth power.

Another disadvantage of the classical measures β_1 and β_2 is that they are only defined on distributions having finite moments. This led many authors to propose alternative measures to overcome the nonrobustness of these classical measures.

The purpose of this paper is twofold; first, to introduce robust alternative measures of skewness and kurtosis that have been developed in the statistics literature, and second, to use some of these measures to study the distribution of Egyptian consumer price changes over the period January 1995 - May 2005.
The paper is structured as follows: Section 2 presents old and new robust measures of skewness. Section 3 introduces old and new robust measures of kurtosis. The distribution of price changes in Egypt is described in section 4 using classical and robust measures of skewness and kurtosis. Concluding remarks are presented in section 5.

2. Robust Measures of Skewness

There are robust measures of location and dispersion that are based on quantiles such as the median and the interquartile range. Following this in 1920, Bowley proposed a coefficient of skewness based on quantiles, which is called the quantile skewness QS:

$$QS = \frac{(Q_{0.75} - Q_{0.5}) - (Q_{0.5} - Q_{0.25})}{Q_{0.75} - Q_{0.25}} \quad (2.1)$$

where $Q_{0.75}$ is the third quantile, $Q_{0.5}$ is the median, and $Q_{0.25}$ is the first quantile. The denominator $Q_{0.75} - Q_{0.25}$ rescales the coefficient so that its maximum value is 1, representing extreme right skewness and its minimum value is -1 for extreme left skewness. The Bowley coefficient has been generalized by Hinkley (1975) to produce the following class of skewness measures:

$$SC = \frac{(Q_{1-p} - Q_{0.5}) - (Q_{0.5} - Q_p)}{Q_{1-p} - Q_p} \quad (2.2)$$

where Q_p $(0 < p < ½)$ is the p^{th} quantile of the random variable X.
The Bowley coefficient of skewness QS (2.1) is a special case of Hinkley's coefficient when $p = 0.25$. The octile skewness OS is produced if $p = 0.125$ in (2.2) as follows:

$$OS = \frac{(Q_{0.875} - Q_{0.5}) - (Q_{0.5} - Q_{0.125})}{Q_{0.875} - Q_{0.125}} \quad (2.3)$$

OS uses more information from the tails of the distribution and can detect asymmetry in the data. Kim and White (2004) conducted extensive Monte Carlo simulations comparing Hinkley's measures with the classical measure β_1, and concluded that β_1 is not a good measure when n is small and a single outlier has a great impact on β_1 compared to Hinkley's measure.
Hosking (1990) formalized the L moments approach which is based on linear combinations of the ordered sample values. Hosking's measure of skewness can be expressed as follows:

$$H = \frac{Q_{0.75} - Q_{0.5}}{Q_{0.5} - Q_{0.25}} \quad (2.4)$$

This measure is interpreted as the ratio of the length of the upper tail to that of the lower tail. H equals1 for symmetric distributions, $H > 1$ for positively skewed distributions, and $H < 1$ for negatively skewed distributions.
Brys et al. (2003) proposed several new measures of skewness which are robust against outlying values. The medcouple measure is produced by replacing some of the quantiles in (2.2) with actual data points:

$$h_1(x_i, x_j) = \frac{(x_{(j)} - Q_{0.5}) - (Q_{0.5} - x_{(i)})}{x_{(j)} - x_{(i)}} \qquad (2.5)$$

with $x_{(i)} < x_{(j)}$

For $x_{(i)} = x_{(j)} = Q_{0.5}$, they set:

$$h_1(x_i, x_j) = \begin{cases} +1 & i > j \\ 0 & i = j \\ -1 & i < j \end{cases}$$

The medcouple is defined as:

$$MC = \text{med}_{x_i \leq Q_{0.5} \leq x_j} \; h_1(x_i, x_j) \qquad (2.6)$$

Next they introduced the medtriple as:

$$MT = \text{med}_{\,x_i < x_j < x_k} \; h_2(x_i, x_j, x_k) \qquad (2.7)$$

where

$$h_2(x_i, x_j, x_k) = \frac{(x_{(k)} - x_{(j)}) - (x_{(j)} - x_{(i)})}{x_{(k)} - x_{(i)}}$$

If $x_{(i)} = x_{(k)}$, they set $h_2(x_i, x_j, x_k) = 0$

Instead of taking the median over all couples or triples of data points, they used a repeated median. The repeated medcouple is defined as:

$$RMC = \text{med}_i \; \text{med}_{j \neq i \;\; x_i \leq Q_{0.5} \leq x_j} \; h_1(x_i, x_j) \qquad (2.8)$$

and the repeated medtriple as:

$$RMT = \text{med}_i \; \text{med}_{j \neq i} \; \text{med}_{k \notin (i, j)} \; h_2(x_i, x_j, x_k) \qquad (2.9)$$

These measures (2.8) and (2.9) are computationally complex.
Groeneveld and Meeden (1984) states the following properties that any reasonable skewness coefficient SC should satisfy:

Property 1. SC is location and scale invariant
i.e. SC (aX+b) = SC (X)

for any $a > 0$, $b \in (-\infty, \infty)$

Property 2: If X is symmetrically distributed around its median, then SC (X) = 0

Property 3: SC (-X) = - SC (X)

Property 4: SC (X) $\in$ [-1, 1]

Groeneveld and Meeden (1984) proved that QS and OS satisfy the mentioned properties. Also, Brys et al. (2003) stated that their new measures (2.6), (2.7), (2.8) and (2.9) satisfy the four mentioned properties. They compared the robustness of these skewness measures towards contamination by using breakdown value which measures the maximum proportion of outliers an estimator can resist without achieving its extreme values. The classical measure β_1 has zero breakdown value whereas QS, MC and RMC have breakdown value of 25%, OS has breakdown value of 12.5%, MT has breakdown value of 20.6% and RMT has a value of 50%. They studied the performance of these measures at symmetric distributions and found that all measures performed well. Concerning the ability to detect small positive skewness, OS is the best, then MC, but QS and RMC do not detect asymmetry adequately. QS was found to be the most insensitive measure to outliers followed by MC.
Brys et al. (2004) studied the influence function of their new measure MC [the influence function measures the relative extent a small perturbation in the distribution has on the measure, Wilcox (1997)]. They found that MC has a bounded influence function in contrast to the classical measure β_1.
The influence functions of QS and OS were also derived by Groeneveld (1991) and are bounded as well. Nothing is known about the breakdown value or the influence function of Hosking's measure H.
From the above discussion of the robust measures of skewness, one can conclude that QS, MC and OS are the best measures, they are all bounded by (-1, 1) and all have positive breakdown values and a bounded influence function. Although the new measures MT, RMC and RMT have good properties, they have large computational complexity, especially in large data sets, and they showed a high sensitivity to outliers at symmetric distributions.
On the other hand, Hosking's measure H is easy to calculate, interpret and robust to outliers, but nothing is known about its breakdown value or its influence function, and it needs further study.

3. Robust Measures of Kurtosis

Karl Pearson introduced the idea of kurtosis to describe distributions that differed from normal distribution in terms of peakedness, but statisticians did not agree on what kurtosis really measures. Modern definitions of kurtosis stress that kurtosis is influenced by both the peakedness and the tail weight of a distribution, and only symmetric distributions should be compared in terms of kurtosis, and it should not be measured by a single number (see Ruppert, 1987; Balanda and MacGillivray, 1988; Groeneveld, 1998; Schmid and Trede, 2003).

A number of robust measures of kurtosis have been proposed. Hogg (1974) found that the following kurtosis coefficient KC which is based on quantiles performed better than β_2 in detecting heavy tailed distributions:

$$KC = \frac{Q_{1-p} - Q_p}{Q_{1-q} - Q_q} \qquad (3.1)$$

where $0 < p < q < 0.5$ and Q_p denotes the p-quantile of the distribution. The problem in using this measure is the choice of p and q and the difference between them. To measure fat tails, p and q should be small to detect the tail behavior of the distribution. Hogg suggested using $q = 0.125$ and $p = 0.025$ in (3.1) which produces T as a measure of fat tails as follows:

$$T = \frac{Q_{0.975} - Q_{0.025}}{Q_{0.875} - Q_{0.125}} \qquad (3.2)$$

T equals 1.7038 for the standard normal distribution and has a low breakdown value of 2.5%.
Following Hogg, Schmid and Trede (2003) suggested P as a measure of peakedness as follows:

$$P = \frac{Q_{0.875} - Q_{0.125}}{Q_{0.75} - Q_{0.25}} \qquad (3.3)$$

which equals 1.7055 for the standard normal distribution. Its breakdown value is 12.5%.
P and T are scale and location invariant and exist for every distribution. As a measure of leptokurtosis, Schmid and Trede (2003) uses L as follows:

$$L = P \times T = \frac{Q_{0.975} - Q_{0.025}}{Q_{0.75} - Q_{0.25}} \qquad (3.4)$$

It equals 2.9058 for the standard normal distribution.
Moores (1988) showed that the classical measure of kurtosis β_2 can be large when probability mass is concentrated either near the mean μ or in the tails of the distribution. He proposed a robust alternative to β_2 which is based on quantiles as follows:

$$M = \frac{(Q_{0.875} - Q_{0.625}) + (Q_{0.375} - Q_{0.125})}{Q_{0.75} - Q_{0.25}} \qquad (3.5)$$

which equals 1.23 for the standard normal distribution.

Brys et al. (2006) proposed tail weight measures based on the robust measures of skewness that are applied to the half of the probability mass lying to the left or the right side of the median of the distribution. They showed that the measures are robust against outlying values and all have a positive breakdown value and a bounded influence function and they are bounded by (-1,1). They can be applied to symmetric as well as asymmetric distributions that do not need to have finite moments. They applied the robust measure (2.2) to the left half ($x < Q_{0.5}$) of the distribution and to the right half ($x > Q_{0.5}$) of the distribution, they obtained the left quantile weight (LQW) and the right quantile weight (RQW) as follows:

$$LQW(p) = -\frac{Q_{(1-p)/2} + Q_{p/2} - 2Q_{0.25}}{Q_{(1-p)/2} - Q_{p/2}} \qquad (3.6)$$

and

$$RQW(q) = \frac{Q_{(1+q)/2} + Q_{(1-q/2)} - 2Q_{0.75}}{Q_{(1+q)/2} - Q_{(1-q/2)}} \qquad (3.7)$$

Where $0 < p < ½$, $½ < q < 1$

(3.6) and (3.7) have breakdown value of 12.5%. Brys et al. (2006) applied the same idea to their medcouple measure to one side of the distribution to obtain the left medcouple (LMC) and to the right medcouple (RMC), defined as:
$LMC = -MC\ (x < Q_{0.5})$ and $RMC = MC\ (x > Q_{0.5})$

LMC and RMC have breakdown value of 12.5%. A distribution will have a positive (negative) right weight RW measure if its upper half (half of the probability larger than the median) is skewed to the right (left). A distribution will have a positive (negative) left weight LW measure if the lower half is skewed to the left (right).
The four measures of tail weight have properties of location and scale invariance and the distribution is symmetric if LW = RW. They can be computed at any distribution even without finite moments.

4. Application: Shape of the Distribution of Price Changes in Egypt

In this section, classical and robust measures of skewness and kurtosis are applied to the bimonthly data of consumer price index (CPI). The data are obtained from the central agency for public mobilization and statistics (CAPMAS) and covers the period (January 1995 – May 2005) with base year 1999/2000, for urban areas, rural areas and total Egypt. This yields a univariate data set consisting of 63 values for each case. The CPI provides a statistical measure of changing levels of retail prices of goods and services, and gives as well a general measure of consumer inflation.
First, the classical and robust measures of skewness and kurtosis are calculated using all 63 observations. Next, outliers in the data are identified using boxplots as shown in Figure 1 from which 9 outliers are identified in the right tail of the distribution. All statistics are recalculated using 54 observations after removing the 9 outliers

(representing 14.3% of the data). This is done to show how the measures perform when outliers are removed from the data.

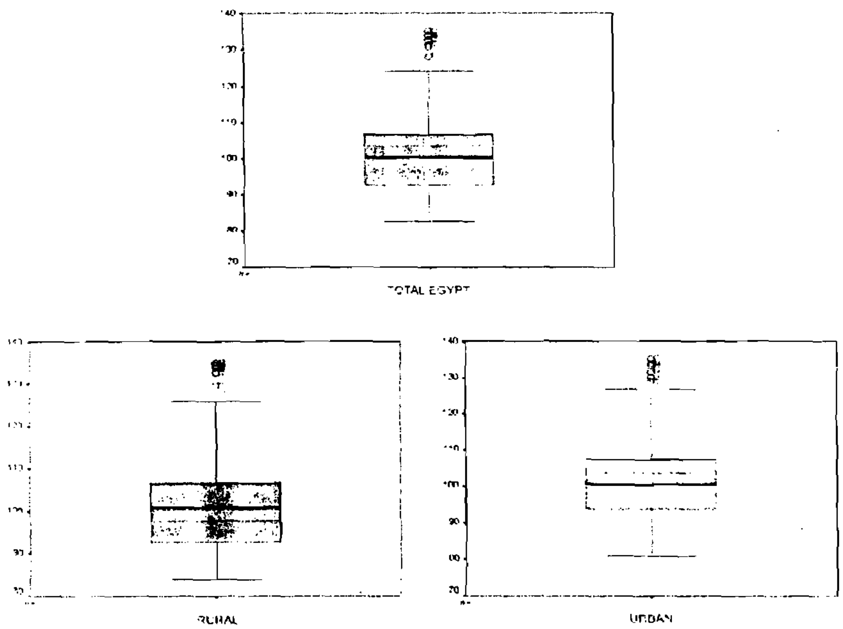

Figure 1 Boxplots for the CPI data for Total Egypt, Rural and Urban

Location measures [mean, median, 5% trimmed mean and Huber's M estimate], scale measures [standard deviation, interquartile range IQR] are calculated as well as skewness measures QS, OS, MC and Hosking's measure H. MT, RMC and RMT are ignored for their computational difficulty. The kurtosis measures used are: T for measuring heavy tails, P for peakedness and L for leptokurtosis, Moores measure M, tail weight measures LQW (0.25) and RQW (0.75), left medcouple (LMC) and right medcouple (RMC).
The boxplots, location and scale measures as well as the required percentiles for the skewness and kurtosis measures are calculated using SPSS 8 package. The medcouple measures are calculated by a simple program using MATHCAD 2001 package. Table 1 reports location and scale measures for the full sample (63 observations) and the reduced sample (54 observations) for total Egypt, urban areas and rural areas. Table 2 reports the skewness measures for the full and reduced sample for the three cases. Table 3 reports the kurtosis measures for the full sample and the reduced sample for the three cases.
From Table 1 we notice that the median is lower than the mean which indicates nonsymmetry of the distribution. After removing the 9 outliers from the data all measures of location become similar and the greatest drop was in the value of the mean and the 5% trimmed mean. As for the measures of scale, the standard deviation

dropped greatly compared to the interquartile range which indicates the sensitivity of the standard deviation to outliers. Measures of location and scale are almost the same for rural and urban areas.

Table 1
Location and scale measures for the CPI data for the full and reduced sample for total Egypt, urban and rural areas.

		Mean	Median	5% Trimmed Mean	Huber's M Estimate	Standard Deviation	IQR
Total Egypt	Full Sample	102.91	100.60	102.25	100.43	14.22	14.40
	Reduced Sample	98.12	98.65	98.01	98.36	8.44	12.42
Urban	Full Sample	102.58	100.60	101.98	100.52	13.95	13.90
	Reduced Sample	97.92	98.20	98.03	98.35	8.35	12.05
Rural	Full Sample	103.15	100.70	102.45	100.39	14.46	14.60
	Reduced Sample	98.27	99.05	97.99	98.19	8.57	12.75

Table 2 shows that the classical measure of skewness β_1 indicates high positive skewness and that the skewness for rural areas is slightly greater than for urban areas, whereas the robust measures OS and MC indicate a lesser degree of positive skewness than β_1. The measures QS and H indicate little negative skewness. After removing the 9 outliers (14.3% of the data), the classical measure β_1 dropped sharply as it appears from the second row for each case which shows that β_1 is dominated by the impact of the 9 large relative price changes in the year 2004 to midyear 2005. OS and MC dropped slightly indicating little negative skewness but QS and H did not change much, which reflects their robustness to outliers.

Table 2
Skewness measures for the CPI data for the full and reduced sample for total Egypt, urban and rural areas.

		β_1	QS	OS	H	MC
Total Egypt	Full Sample	0.935	-0.083	0.351	0.846	0.104
	Reduced Sample	0.008	-0.123	-0.155	0.781	-0.101
Urban	Full Sample	0.880	-0.007	0.328	0.986	0.065
	Reduced Sample	-0.284	-0.025	-0.108	0.979	-0.008
Rural	Full Sample	0.970	-0.151	0.365	0.738	0.119
	Reduced Sample	0.234	-0.200	-0.196	0.667	-0.186

Table 3 illustrates that the classical and robust measures of kurtosis reflect excess kurtosis which means that the distribution is leptokurtic, i.e. more sharply peaked than the normal and has fat tail except for the measure T which indicates thinner tails than the normal. It is noticed that β_2 underestimates the value of kurtosis which means that it underestimates the importance of outliers. It is noticed that the kurtosis for urban areas is a little bigger than for rural except for the M measure.
The four measures of tail weight indicate the heaviness of the right tail of the distribution. After removing the outliers (14.3% of the data) value of β_2 changed greatly indicating slight platykurtosis. The same happened to the robust measures due to their low breakdown values (12.5%) and the large proportions of outliers (14.3%). The left tail weight measures did not change much except for rural areas. The right tail measures naturally changed after removing 14.3% of the observations due to the fact that its breakdown value is only 12.5%. The QW measures give better idea of tail weight especially after removing the outliers from the right tail of the distribution.
LQW (.25) became larger than RQW (.75) in the three cases whereas LMC does not reflect this picture.

Table 3
Kurtosis measures for the CPI data for the full and reduced sample for total Egypt, urban and rural areas.

		β_2	T	P	L	M	LQW(.25)	RQW(.75)	LMC	RMC
Total Egypt	Full Sample	0.208	1.257	2.847	3.579	2.417	0.089	0.708	0.132	0.673
	Reduced Sample	-0.604	1.613	1.652	2.664	1.069	0.234	0.196	0.152	0.236
Urban	Full Sample	0.324	1.349	2.827	3.813	2.338	0.305	0.670	0.299	0.604
	Reduced Sample	-0.622	1.492	1.749	2.609	1.261	0.202	0.165	0.233	0.209
Rural	Full Sample	0.140	1.197	2.890	3.459	2.493	-0.048	0.745	0.014	0.712
	Reduced Sample	-0.396	1.732	1.558	2.698	0.894	0.300	0.179	0.050	0.250

Note: T for N (0,1)= 1.7038, P for N (0,1)= 1.7055, L for N (0,1)= 2.9058, M for N (0,1)= 1.230

5. Concluding Remarks

In this paper the properties of the distribution of Egyptian consumer price changes have been studied by using classical and robust measures of locations, scale, skewness and kurtosis. The robust measures of skewness showed that the distribution is asymmetric with little right skewness and mild kurtosis.
The tail weight measures showed the tendency for the right tail of the distribution to dominate the left tail. The distribution for rural areas is more skewed to the right than

that for urban areas and the peakedness is almost the same with u-shaped left tail for the distribution of rural areas.

It is noticed that the measure β_1 overestimates skewness. It is sensitive to outliers. On the contrary, all robust measures of skewness are not heavily influenced by the large fraction of outliers. The measures OS and MC are preferred than measures QS and H although the latter measures are robust to outliers but OS uses more information from the tails of the distribution and MC uses the majority of the data set.

Concerning the measures of kurtosis, it is clear that β_2 underestimates kurtosis and the robust measure T gives contradictory results compared to the other robust measures.

It is evident that there is no clear definite evidence on what robust measure of kurtosis is best, but it is recommended to use the tail weight measures especially the QW measures for the simplicity of their calculation. The tail weight measures provide additional descriptive information regarding the shape of the distribution. Finally, it is clear that the classical measures β_1 and β_2 are not robust to outliers, but there is a need for further investigation to decide which robust measures are the best.

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